

SPM BULLETIN

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1. EDITOR'S NOTE

The slides of the talks given at the conference *Ultramath* (Pisa, Italy, June 2008) are available at

<http://www.dm.unipi.it/~ultramath/abstracts.html>

Enjoy.

Boaz Tsaban, tsaban@math.biu.ac.il
<http://www.cs.biu.ac.il/~tsaban>

2. RESEARCH ANNOUNCEMENTS

2.1. Combinatorial and model-theoretical principles related to regularity of ultrafilters and compactness of topological spaces, I. We begin the study of the consequences of the existence of certain infinite matrices. Our present application is to compactness of products of topological spaces.

<http://arxiv.org/abs/0803.3498>

Paolo Lipparini

2.2. Fréchet-Urysohn fans in free topological groups. In this paper we answer the question of T. Banach and M. Zarichnyi constructing a copy of the Fréchet-Urysohn fan S_ω in a topological group G admitting a functorial embedding $[0, 1] \subset G$. The latter means that each autohomeomorphism of $[0, 1]$ extends to a continuous homomorphism of G . This implies that many natural free topological group constructions (e.g. the constructions of the Markov free topological group, free abelian topological group, free totally bounded group, free compact group) applied to a Tychonov space X containing a topological copy of the space \mathbb{Q} of rationals give topological groups containing S_ω .

<http://arxiv.org/abs/0803.4117>

Taras Banach, Dušan Repovš, and Lyubomyr Zdomskyy

2.3. Packing index of subsets in Polish groups. For a subset A of a Polish group G , we study the (almost) packing index $\text{ind}_P(A)$ (resp. $\text{Ind}_P(A)$) of A , equal to the supremum of cardinalities $|S|$ of subsets $S \subset G$ such that the family of shifts $\{xA\}_{x \in S}$ is (almost) disjoint (in the sense that $|xA \cap yA| < |A|$ for any distinct points $x, y \in S$). Subsets $A \subset G$ with small (almost) packing index are small in a geometric sense. We show that $\text{ind}_P(A) \in \mathbb{N} \cup \{\aleph_0, \mathfrak{c}\}$ for any σ -compact subset A of a Polish group. If $A \subset G$ is Borel, then the packing indices $\text{ind}_P(A)$ and $\text{Ind}_P(A)$ cannot take values in the half-interval $[\mathfrak{sq}(\Pi_1^1), \mathfrak{c})$ where $\mathfrak{sq}(\Pi_1^1)$ is a certain uncountable cardinal that is smaller than \mathfrak{c} in some models of ZFC. In each non-discrete Polish Abelian group G we construct two closed subsets $A, B \subset G$ with $\text{ind}_P(A) = \text{ind}_P(B) = \mathfrak{c}$ and $\text{Ind}_P(A \cup B) = 1$ and then apply this result to show that G contains a nowhere dense Haar null subset $C \subset G$ with $\text{ind}_P(C) = \text{Ind}_P(C) = \kappa$ for any given cardinal number $\kappa \in [4, \mathfrak{c}]$.

<http://arxiv.org/abs/0804.1333>

Taras Banach, Nadya Lyaskovska, and Dušan Repovš

2.4. Symmetric monochromatic subsets in colorings of the Lobachevsky plane. We prove that for each partition of the Lobachevsky plane into finitely many Borel pieces one of the cells of the partition contains an unbounded centrally symmetric subset.

<http://arxiv.org/abs/0804.1335>

T. Banach, A. Dudko and D. Repovš

2.5. Structural Ramsey theory of metric spaces and topological dynamics of isometry groups. In 2003, Kechris, Pestov and Todorćević showed that the structure of certain separable metric spaces - called ultrahomogeneous - is closely related to the combinatorial behavior of the class of their finite metric spaces. The purpose of the present paper is to explore the different aspects of this connection.

<http://arxiv.org/abs/0804.1593>

L. Nguyen Van Thé

2.6. Distinguishing Number of Countable Homogeneous Relational Structures. The distinguishing number of a graph G is the smallest positive integer r such that G has a labeling of its vertices with r labels for which there is no non-trivial automorphism of G preserving these labels. Albertson and Collins computed the distinguishing number for various finite graphs, and Imrich, Klavžar and Trofimov computed the distinguishing number of some infinite graphs, showing in particular that the Random Graph has distinguishing number 2. We compute the distinguishing number of various other finite and countable homogeneous structures, including undirected and directed graphs, and posets. We show that this number is in most cases two or infinite, and besides a few exceptions conjecture that this is so for all primitive homogeneous countable structures.

<http://arxiv.org/abs/0804.4019>

C. Laflamme, L. Nguyen Van Thé, N. W. Sauer

2.7. Indestructible colourings and rainbow Ramsey theorems. We give a negative answer to a question of Erdős and Hajnal: it is consistent that GCH holds and there is a colouring $c : [\omega_2]^2 \rightarrow 2$ establishing $\omega_2 \not\rightarrow [(\omega_1; \omega)]_2^2$ such that some colouring $g : [\omega_1]^2 \rightarrow 2$ can not be embedded into c . It is also consistent that 2^{ω_1} is arbitrarily large, and a function g establishes $2^{\omega_1} \not\rightarrow [(\omega_1, \omega_2)]_{\omega_1}^2$ such that there is no uncountable g -rainbow subset of 2^{ω_1} . We also show that for each $k \in \omega$ it is consistent with Martin's Axiom that the negative partition relation $\omega_1 \not\rightarrow^* [(\omega_1; \omega_1)]_{k-bdd}$ holds.

<http://arxiv.org/abs/0804.4548>

Lajos Soukup

2.8. Products of Borel subgroups. We investigate the Borelness of the product of two Borel subgroups in Polish groups. While the intersection of these two subgroups is Polishable, the Borelness of their product is confirmed. On the other hand, we construct two Δ_3^0 subgroups whose product is not Borel in every uncountable abelian Polish group.

www.ams.org/proc/0000-000-00/S0002-9939-08-09334-9

Longyun Ding and Bingqing Li

2.9. Selection theorems and treeability. We show that domains of non-trivial Σ_1^1 trees have Δ_1^1 members. Using this, we show that smooth treeable equivalence relations have Borel transversals, and essentially countable treeable equivalence relations have Borel complete countable sections. We show also that treeable equivalence relations which are ccc idealistic, measured, or generated by a Borel action of a Polish group have Borel complete countable sections.

<http://www.ams.org/journal-getitem?pii=S0002-9939-08-09548-8>

Greg Hjorth

2.10. Combinatorial and model-theoretical principles related to regularity of ultrafilters and compactness of topological spaces, IV. We extend to singular cardinals the model-theoretical relation $\lambda \xrightarrow{\kappa} \mu$ introduced in P. Lipparini, The compactness spectrum of abstract logics, large cardinals and combinatorial principles, Boll. Unione Matematica Italiana ser. VII, **4-B** 875–903 (1990). We extend some results obtained in Part II, finding equivalent conditions involving uniformity of ultrafilters and the existence of certain infinite matrices. Our present definition suggests a new compactness property for abstract logics.

<http://arxiv.org/abs/0805.1548>

Paolo Lipparini

2.11. A property of $C_p[0, 1]$. We prove that for every finite dimensional compact metric space X there is an open continuous linear surjection from $C_p[0, 1]$ onto $C_p(X)$. The proof makes use of embeddings introduced by Kolmogorov and Sternfeld in connection with Hilbert's 13th problem.

<http://arxiv.org/abs/0806.2719>

Michael Levin

2.12. A Dedekind Finite Borel Set. In this paper we prove three theorems about the theory of Borel sets in models of ZF without any form of the axiom of choice. We prove that if B is a $G_{\delta\sigma}$ set, then either B is countable or B contains a perfect subset. Second, we prove that if the real line is the countable union of countable sets, then there exists an $F_{\sigma\delta}$ set which is uncountable but contains no perfect subset. Finally, we construct a model of ZF in which we have an infinite Dedekind finite set of reals which is $F_{\sigma\delta}$.

<http://www.math.wisc.edu/~miller/res/ded.pdf>

Arnold W. Miller

2.13. Aronszajn Compacta. We consider a class of compacta X such that the maps from X onto metric compacta define an Aronszajn tree of closed subsets of X .

<http://arxiv.org/abs/0806.4499>

Joan E. Hart and Kenneth Kunen

2.14. A strong antidiomond principle compatible with CH. A strong antidiomond principle $(*c)$ is shown to be consistent with CH. This principle can be stated as a “ P -ideal dichotomy”: every P -ideal on $\omega_1 - 1$ (i.e. an ideal that is σ -directed under inclusion modulo finite) either has a closed unbounded subset of ω_1 locally inside of it, or else has a stationary subset of ω_1 orthogonal to it. We rely on Shelah’s theory of parameterized properness for NNR iterations, and make a contribution to the theory with a method of constructing the properness parameter simultaneously with the iteration. Our handling of the application of the NNR iteration theory involves definability of forcing notions in third order arithmetic, analogous to Souslin forcing in second order arithmetic.

<http://arxiv.org/abs/0806.4220>

James Hirschorn

2.15. On the strength of Hausdorff’s gap condition. Hausdorff’s gap condition was satisfied by his original 1936 construction of an (ω_1, ω_1) gap in $P(\mathbb{N})/Fin$. We solve an open problem in determining whether Hausdorff’s condition is actually stronger than the more modern indestructibility condition, by constructing an indestructible (ω_1, ω_1) gap not equivalent to any gap satisfying Hausdorff’s condition, from uncountably many random reals.

<http://arxiv.org/abs/0806.4732>

James Hirschorn

2.16. Nonhomogeneous analytic families of trees. We consider a dichotomy for analytic families of trees stating that either there is a colouring of the nodes for which all but finitely many levels of every tree are nonhomogeneous, or else the family contains an uncountable antichain. This dichotomy implies that every nontrivial Souslin poset satisfying the countable chain condition adds a splitting real. We then reduce the dichotomy to a conjecture of Sperner Theory. This conjecture is concerning the asymptotic behaviour of the product of the sizes of the m -shades of pairs of cross- t -intersecting families.

<http://arxiv.org/abs/0807.0147>

James Hirschorn

2.17. Reasonable non-Radon-Nikodym ideals. For any abelian Polish σ -compact group H there exist a σ -ideal Z over \mathbb{N} and a Borel Z -approximate homomorphism $f : H \rightarrow H^{\mathbb{N}}$ which is not Z -approximable by a continuous true homomorphism $g : H \rightarrow H^{\mathbb{N}}$.

<http://arxiv.org/abs/0806.4760>

Vladimir Kanovei and Vassily Lyubetsky

2.18. σ -continuity and related forcings. The Steprans forcing notion arises as a quotient of Borel sets modulo the ideal of σ -continuity of a certain Borel not σ -continuous function. We give a characterization of this forcing in the language of trees and using this characterization we establish such properties of the forcing as fusion and continuous reading of names. Although the latter property is usually implied by the fact that the associated ideal is generated by closed sets, we show it is not the case with Steprans forcing. We also establish a connection between Steprans forcing and Miller forcing thus giving a new description of the latter. Eventually, we exhibit a variety of forcing notions which do not have continuous reading of names in any presentation.

<http://arxiv.org/abs/0807.1254>

Marcin Sabok

2.19. An exact Ramsey principle for block sequences. We prove an exact, i.e., formulated without Δ -expansions, Ramsey principle for infinite block sequences in vector spaces over countable fields, where the two sides of the dichotomic principle are represented by respectively winning strategies in Gowers' block sequence game and winning strategies in the infinite asymptotic game. This allows us to recover Gowers' dichotomy theorem for block sequences in normed vector spaces by a simple application of the basic determinacy theorem for infinite asymptotic games.

<http://arxiv.org/abs/0807.2205>

Christian Rosendal

2.20. Baire reflection. We study reflection principles involving nonmeager sets and the Baire Property which are consequences of the generic supercompactness of ω_2 , such as the principle asserting that any point countable Baire space has a stationary set of closed subspaces of weight ω_1 which are also Baire spaces. These principles entail the analogous principles of stationary reflection but are incompatible with forcing axioms. Assuming MM, there is a Baire metric space in which a club of closed subspaces of weight ω_1 are meager in themselves. Unlike stronger forms of Game Reflection, these reflection principles do not decide CH, though they do give ω_2 as an upper bound for the size of the continuum.

<http://www.ams.org/tran/0000-000-00/S0002-9947-08-04503-0/home.html>

Stevo Todorćević and Stuart Zoble

2.21. Tukey classes of ultrafilters on ω . Motivated by a question of Isbell, we show that Jensen's Diamond Principle implies there is a non-P-point ultrafilter U on ω such that U , whether ordered by reverse inclusion or reverse inclusion mod finite, is not Tukey equivalent to the finite sets of reals ordered by inclusion. We also show that, for every regular infinite κ not greater than 2^{\aleph_0} , if $MA(\sigma\text{-centered})$

holds, then some ultrafilter U on ω , ordered by reverse inclusion mod finite, is Tukey equivalent to the sets of reals of size less than κ , ordered by inclusion. We also prove two negative ZFC results about the possible Tukey classes of ultrafilters on ω .

<http://arxiv.org/abs/0807.3978>

David Milovich

2.22. Countably determined compact abelian groups. For an abelian topological group G let \widehat{G} be the dual group of all continuous characters endowed with the compact open topology. Given a closed subset X of an infinite compact abelian group G such that $w(X) < w(G)$ and an open neighbourhood U of 0 in T , we show that $|\{\pi \in \widehat{G} : \pi(X) \subseteq U\}| = |\widehat{G}|$. (Here $w(G)$ denotes the weight of G .) A subgroup D of G determines G if the restriction homomorphism $\widehat{G} \rightarrow \widehat{D}$ of the dual groups is a topological isomorphism. We prove that $w(G) = \min\{|D| : D \text{ is a subgroup of } G \text{ that determines } G\}$ for every compact abelian group G . In particular, an infinite compact abelian group determined by its countable subgroup must be metrizable. This gives a negative answer to questions of Comfort, Hernández, Macario, Raczkowski and Trigos-Arrieta. As an application, we furnish a short elementary proof of the result that compact determined abelian groups are metrizable.

<http://arxiv.org/abs/0807.3846>

Dikran Dikranjan, Dmitri Shakhmatov

2.23. A topological reflection principle equivalent to Shelah's Strong Hypothesis. We notice that Shelah's Strong Hypothesis is equivalent to the following reflection principle: Suppose $\langle X, \tau \rangle$ is a first-countable space whose density is a regular cardinal, κ . If every separable subspace of X is of cardinality at most κ , then the cardinality of X is κ .

dx.doi.org/10.1090/S0002-9939-08-09411-2

Assaf Rinot

2.24. Superfilters, Ramsey theory, and van der Waerden's Theorem. Superfilters are generalized ultrafilters, which capture the underlying concept in Ramsey theoretic theorems such as van der Waerden's Theorem. We establish several properties of superfilters, which generalize both Ramsey's Theorem and its variant for ultrafilters on the natural numbers. We use them to confirm a conjecture of Kočinac and Di Maio, which is a generalization of a Ramsey theoretic result of Scheepers, concerning selections from open covers. Following Bergelson and Hindman's 1989 Theorem, we present a new simultaneous generalization of the theorems of Ramsey, van der Waerden, Schur, Folkman-Rado-Sanders, Rado, and others, where the colored sets can be much smaller than the full set of natural numbers.

<http://arxiv.org/abs/0808.1654>

Nadav Samet and Boaz Tsaban

3. UNSOLVED PROBLEMS FROM EARLIER ISSUES

Issue 1. *Is $\left(\frac{\Omega}{\Gamma}\right) = \left(\frac{\Omega}{\mathbb{T}}\right)$?*

Issue 2. *Is $U_{\text{fin}}(\mathcal{O}, \Omega) = S_{\text{fin}}(\Gamma, \Omega)$? And if not, does $U_{\text{fin}}(\mathcal{O}, \Gamma)$ imply $S_{\text{fin}}(\Gamma, \Omega)$?*

Issue 4. *Does $S_1(\Omega, \mathbb{T})$ imply $U_{\text{fin}}(\Gamma, \Gamma)$?*

Issue 5. *Is $\mathfrak{p} = \mathfrak{p}^*$? (See the definition of \mathfrak{p}^* in that issue.)*

Issue 6. *Does there exist (in ZFC) an uncountable set satisfying $S_{\text{fin}}(\mathcal{B}, \mathcal{B})$?*

Issue 8. *Does $X \notin \text{NON}(\mathcal{M})$ and $Y \notin \text{D}$ imply that $X \cup Y \notin \text{COF}(\mathcal{M})$?*

Issue 9 (CH). *Is $\text{Split}(\Lambda, \Lambda)$ preserved under finite unions?*

Issue 10. *Is $\text{cov}(\mathcal{M}) = \mathfrak{od}$? (See the definition of \mathfrak{od} in that issue.)*

Issue 11. *Does $S_1(\Gamma, \Gamma)$ always contain an element of cardinality \mathfrak{b} ?*

Issue 12. *Could there be a Baire metric space M of weight \aleph_1 and a partition \mathcal{U} of M into \aleph_1 meager sets where for each $\mathcal{U}' \subset \mathcal{U}$, $\bigcup \mathcal{U}'$ has the Baire property in M ?*

Issue 14. *Does there exist (in ZFC) a set of reals X of cardinality \mathfrak{d} such that all finite powers of X have Menger's property $S_{\text{fin}}(\mathcal{O}, \mathcal{O})$?*

Issue 15. *Can a Borel non- σ -compact group be generated by a Hurewicz subspace?*

Issue 16 (MA). *Is there an uncountable $X \subseteq \mathbb{R}$ satisfying $S_1(\mathcal{B}_\Omega, \mathcal{B}_\Gamma)$?*

Issue 17 (CH). *Is there a totally imperfect X satisfying $U_{\text{fin}}(\mathcal{O}, \Gamma)$ that can be mapped continuously onto $\{0, 1\}^{\mathbb{N}}$?*

Issue 18 (CH). *Is there a Hurewicz X such that X^2 is Menger but not Hurewicz?*

Issue 19. *Does the Pytkeev property of $C_p(X)$ imply that X has Menger's property?*

Issue 20. *Does every hereditarily Hurewicz space satisfy $S_1(\mathcal{B}_\Gamma, \mathcal{B}_\Gamma)$?*

Issue 21 (CH). *Is there a Rothberger-bounded $G \leq \mathbb{Z}^{\mathbb{N}}$ such that G^2 is not Menger-bounded?*

Issue 22. *Let \mathcal{W} be the van der Waerden ideal. Are \mathcal{W} -ultrafilters closed under products?*

Issue 23. *Is the δ -property equivalent to the γ -property $\left(\frac{\Omega}{\Gamma}\right)$?*

Previous issues. The previous issues of this bulletin are available online at <http://front.math.ucdavis.edu/search?&t=%22SPM+Bulletin%22>

Contributions. Announcements, discussions, and open problems should be emailed to tsaban@math.biu.ac.il

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